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R/C BUILDING Software

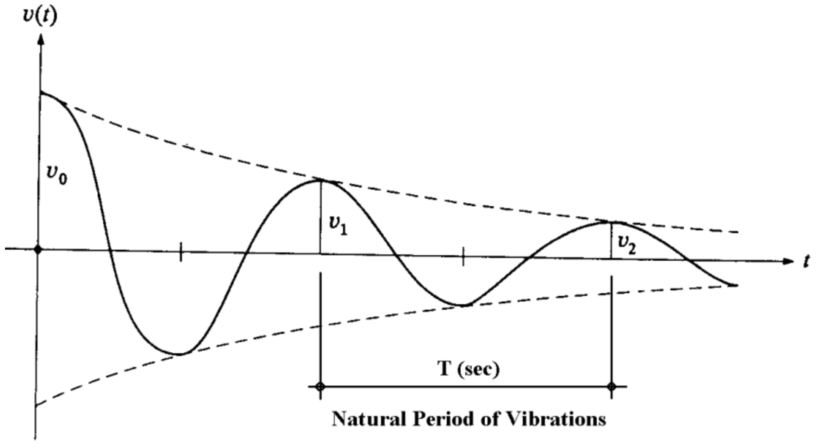
Dynamic Analysis: Frequency and Spectral

Summary

R/C BUILDING software can be used to evaluate the Natural Periods of a building's structural system and to perform a Spectral Analysis. The software has three alternative methods to perform the Frequency and Spectral analysis.

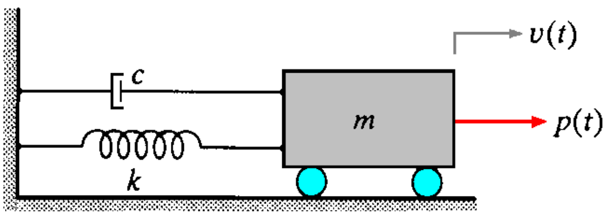
What is Frequency Analysis

Frequency analysis is an analytical procedure, which uses the stiffness and mass properties of a structure and to evaluate the dynamic properties of a building structure. The dynamic properties of a structure are defined as a series of mode shapes and associated natural period (T, seconds) or frequencies (f, Hz). Note that f = 1 / T.



Natural Period of Free Vibrations of a Simple Oscillator

In the figure above the displacement over time of a simple oscillator is shown. The oscillator was "forced" to vibrate by some external action, and then let to vibrate freely. The length of the sin wave of the free vibrations is the natural period (T). The natural period of the free vibrations is a function of the stiffness (k) and the mass (m) of the structure.



Simple Oscillator, SDFS (Single Degree of Freedom System)

The Natural Periods (T) of the free vibrations are properties of the structure, and they can be used to evaluate how the structure will behave (respond) if subjected to dynamic loading. Dynamic loading is loading which varies over time, such as wind, earthquake, rotating machinery and explosions. The dynamic loading will act on the structure with varying magnitude over a certain length of time, during which the structure will vibrate, and the mass of the structure will "receive" some acceleration that will produce inertial forces (Mass * Acceleration = Force). When the structure deforms it will generate internal elastic forces. The vibrations will also cause friction between the material particles, and between the structural components, which will generate damping forces. All these forces: the external dynamic force (action), internal elastic forces, inertia (Mass * Acceleration), and damping force, must be in equilibrium at any time.

$$f_I(t) + f_D(t) + f_S(t) = p(t)$$

$$f_I(t) = m\,\ddot{v}(t) \quad \text{inertial force}$$
$$f_D(t) = c\,\dot{v}(t) \quad \text{viscous damping}$$
$$f_S(t) = k\,v(t) \quad \text{elastic force}$$

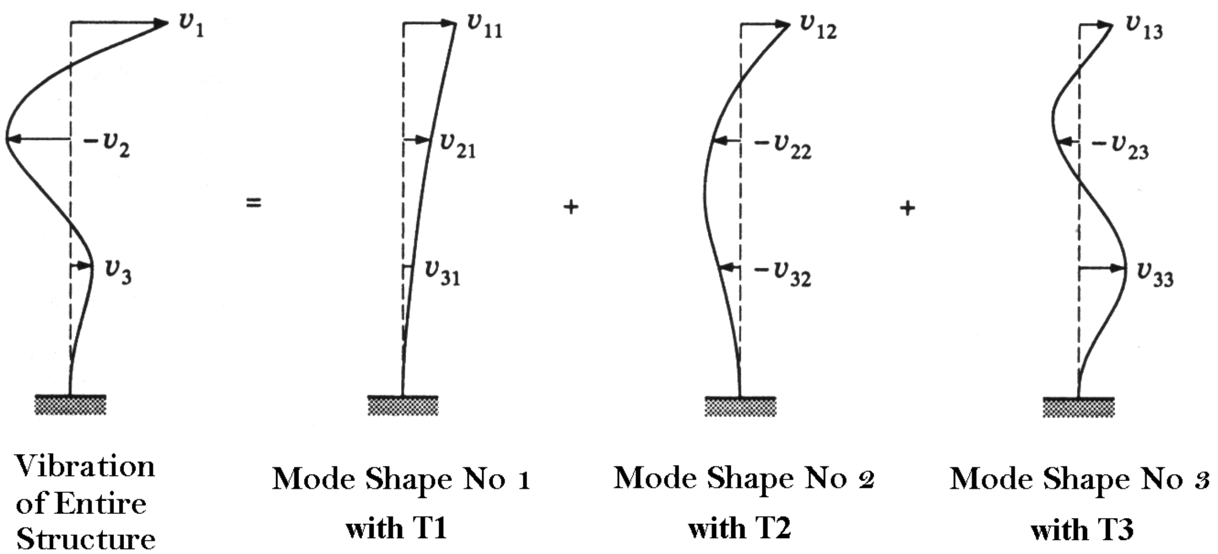
$$m\,\ddot{v}(t) + c\,\dot{v}(t) + k\,v(t) = p(t)$$
equation of motion

displacement $v(t)$
applied load $p(t)$

Equitation of Vibrations

The Natural Periods can be used to assess the sensitivity of a structure subjected to dynamic loading, and to evaluate the risk of excessive vibrations. For instance, if some of the mode shapes of the structure have similar frequencies as vibrating machinery attached to the structure, it is very likely that this will cause excessive vibrations due to the resonance effect.

The other use of the mode shapes is in the Dynamic Response Analysis of the structure. The vibrations of the structure due to some external dynamic loading can be evaluated as a sum of vibrations of simple oscillators (SDFS, Single Degree of Freedom Systems). The figure below shows that at any time during the vibrations, the deformed shape of the structure (on the left) can be expressed as a sum of several mode shapes. The amplitude of each mode shape is evaluated as a SDFS subjected to the dynamic force. This procedure is called the Mode Superposition Method.

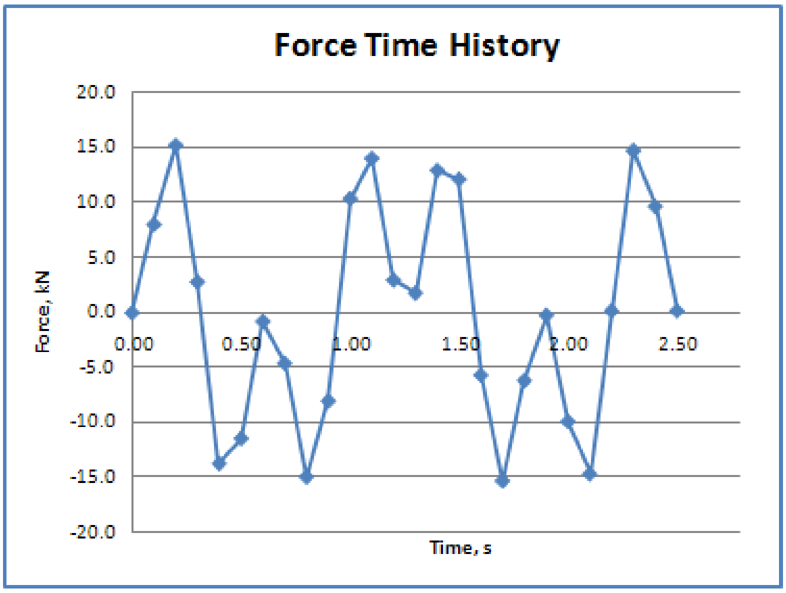


Mode Superposition

What is Spectral Analysis

A dynamic force can expressed as a time dependant function, Time History. In the figure below a dynamic force is plotted over a total period of 2.5 seconds at regular intervals of 0.1 seconds.

t s	force kN
0.00	0.0
0.10	8.1
0.20	15.3
0.30	2.8
0.40	-13.8
0.50	-11.5
0.60	-0.8
0.70	-4.6
0.80	-15.0
0.90	-8.1
1.00	10.4
1.10	14.1
1.20	3.0
1.30	1.8
1.40	13.0
1.50	12.2
1.60	-5.7
1.70	-15.4
1.80	-6.2
1.90	-0.2
2.00	-9.9
2.10	-14.7
2.20	0.2
2.30	14.8
2.40	9.7
2.50	0.2



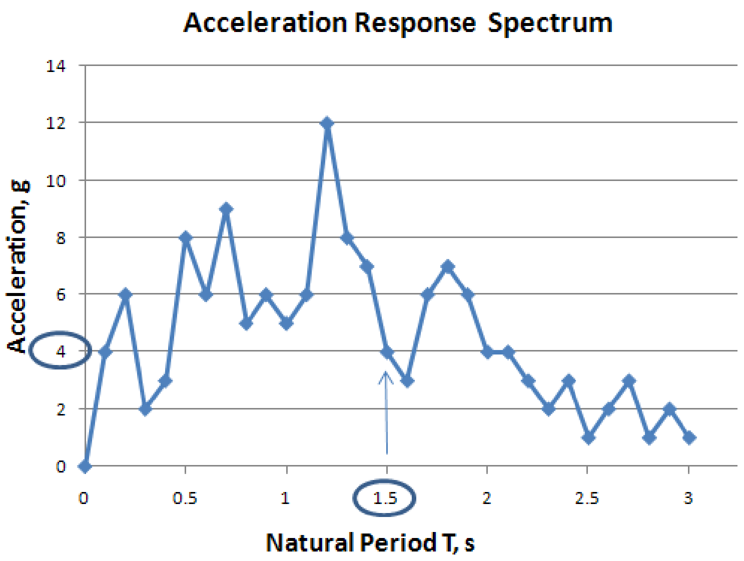
Time History of Dynamic Force

If this force is applied to a point on a structure it will cause the structure to vibrate, i.e. the structure will "respond". At any point of the structure a similar time history of the displacement, and any other internal forces will be generated. The displacements and the internal forces generated by an external dynamic load is called the "Dynamic Response". This type of response as a time history output is not practical as it is very hard to follow the changes over a period of time. The results of interest are the maximum values of the displacement and the internal forces. We can then implement the Spectral Analysis approach which will provide only the maximum values of the displacement and the internal forces. In order to implement the Spectral Method, the dynamic force given as the Time History has to be transformed into a Response Spectrum curve.

A Response Spectrum curve is devised in the following manner:

> The dynamic force given as a time history (see figure above) is applied to a simple oscillator (SDFS, Single Degree of Freedom System) defined with its natural period T.

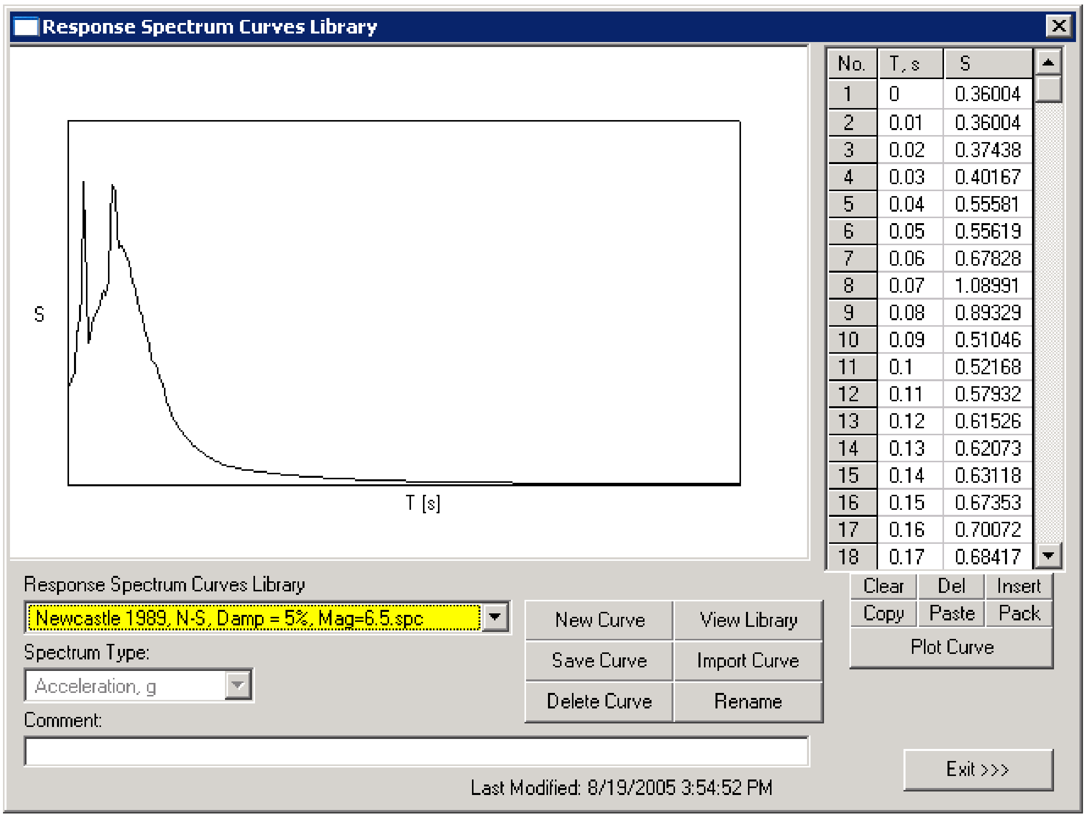
The SDFS will vibrate and a time history of the displacements and the acceleration can be calculated. We can then select the maximum acceleration of the SDFS. For example, if we consider a SDFS with period T=1.5s and maximum acceleration equal to 4 g. (g is earth acceleration) we can use different natural periods from zero to 3s at regular intervals of 0.1s. We can then develop the complete Spectral Response curve (see figure below). Note that the simple oscillator (SDFS) has some nominal dumping, and normally this damping is assumed to be 5%.



Spectral Curve

The spectral curve is called a Response Spectrum because the spectral values (acceleration in this case) are the maximum acceleration values of the response obtained on a SDFS. In a similar manner we can derive Response Spectral Curves for displacement and velocity. However it is customary to use the Acceleration Response Spectrum curve.

These types of Acceleration Response Spectrum curves are given in any earthquake design codes, such as AS 1170.4. The earthquake action is represented with a typical Acceleration Response Spectrum curve. In the design codes several averaged Spectral Response Curves are provided for different soil conditions. These curves are scaled to adjust for different levels of earthquakes. The spectral curves can be specific for particular seismic events. (see figure below)



Spectral Curve of a Specific Seismic Event

For larger buildings or for buildings of greater significance, a location-specific Spectral Curves should be developed. Based on the seismic study of the specific site and the local soil conditions, a synthetic Spectral Curve can be produced. These location-specific synthetic Spectral Curves will represent the seismic events most accurately.

The Spectral Analysis is performed in the following manner:

-Perform the Frequency Analysis and evaluate several mode shapes and the associated natural periods, T1, T2, T3, etc..
-Obtain an Acceleration Response Spectrum curve, which represents a particular dynamic load.
-For each natural period (T) evaluate the spectral acceleration, using the Spectral curve.
-Evaluate the participation of each mode shape in the dynamic response of the structure using the corresponding spectral acceleration and participation factors. For each mode shape the maximum displacements is calculated.
-Combine the modal displacements to obtain the maximum displacement of the structure.
-Use this displacement to obtain the internal forces.

R/C BUIDLING Software: Three Alternative Methods for Dynamic Analysis

R/C BUILDING software offers three alternative methods do evaluate the mode shapes and natural periods, and to perform Spectral Analysis:

2D Simple
2D Advanced
3D Complete

This is a brief description of the three methods:

2D Simple

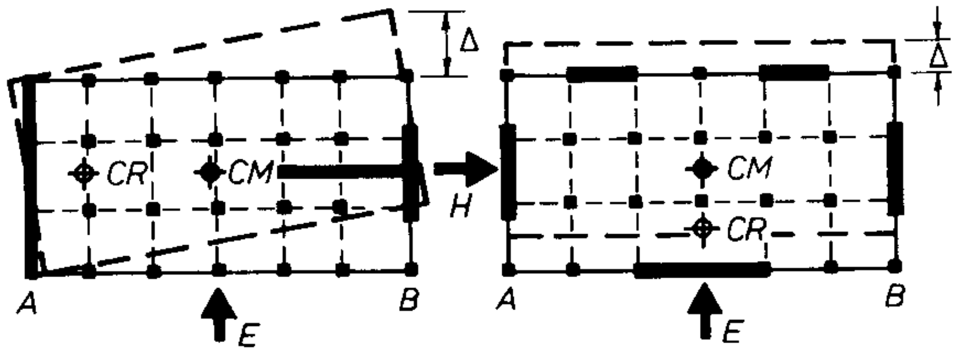
The dominant mode shapes in X-X and Y-Y are evaluated reliably. An initial set of lateral forces are required in order to evaluate the mode shapes (Higher mode shapes are estimated, and the rotational mode shapes are neglected).

This method may be used for regular structures, or only when the dominant mode shapes in X-X and Y-Y are of interest. This method can also be used to confirm and check the dominant periods obtained by the more advanced methods.

2D Advanced

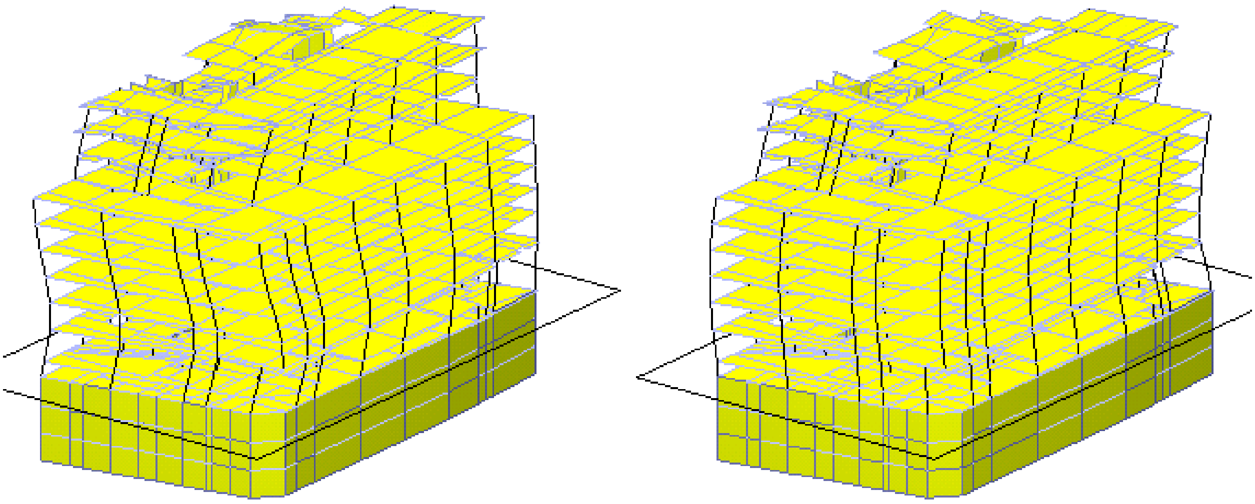
The first several mode shapes in X-X and Y-Y are evaluated reliably. No initial lateral forces are needed to evaluate the mode shapes. This approach is reliable for relatively regular structures. The rotational mode shapes are neglected.

In both 2D approaches defined above, the structure can rotate about the vertical axes but the rotational modes shapes are not evaluated. The translation of the each level is represented as an average of the four floor corners. If the floor rotation is significant then the translational mode shapes are not very reliably presented. (see figure below)



Global Torsion

3D Compete
All mode shapes, translation in X-X and Y-Y, and rotation about the vertical axes are evaluated reliably. If sufficient number of mode shapes are evaluated the Mass Participation Factor in each orthogonal direction will be greater than 90%. This is the most reliable method for dynamic analysis suitable for any structure.



Example: 3D Mode Shapes

Usage: Frequency and Spectral, 2D Simple

Prepare a 3D model of the building structural system using the R/C Building software program. The model needs to contain all the geometry and all vertical loading.

The model should have a minimum of 4 basic Load Cases, for example:

1. G (dead load)
2. Q (live load)
3. Ex (earthquake force in the X-X direction)
4. Ey (earthquake force in the Y-Y direction)

The model must have a Service Load Combination that will be used to evaluate the mass of the structure and then incorporated in the frequency analysis. For instance, we can define one load combination with 100% Deal Load and 40% Live Load. In this case the load factors are: 1.0G + 0.4Q

On the Model and Solver Settings input panel select:
Load Combination for Floor Mass Centroid: 1.0G + 0.4Q

Select:
Load Case 3: Earthquake in X-X
Load Case 4: Earthquake in Y-Y

Select the number of mode shapes (3 to 6 should be sufficient). Then select the "Simple 2D" option for evaluating the mode shapes.

On the Lateral Load Earthquake input panel, set the lateral earthquake forces in X-X and Y-Y as Basic Load Cases 3 and 4. The magnitude of the lateral forces should be at least 2.5% base shear. This is the initial set of lateral forces needed to evaluate the mode shapes.

Mesh and analyse the model using the Linear Static Analysis option.

Select [Solve > Frequency Analysis]. A text report will open showing the frequency results.

The mode shapes and natural periods shown in the report will now be used for the Spectral Analysis (in the Earthquake Lateral Load input panel).

In the [Input > Lateral Load] window, select basic "Load Case: 3. Ex" and press on the "Earthquake AS 1170.4" button followed by selecting "Dynamic Analysis". Select "X-X direction" and enter all input parameters and press OK. This will replace the initial set of lateral forces in the table for the X-X direction.
Select "Load Case: 4. Ey" and repeat the procedure outlined in the step above. These lateral forces are evaluated by the simple 2D Dynamic Spectral method.

The model is now reanalyzed by the 2nd Order Static Analysis option and examine the results can be examined.

The Earthquake load calculator allows usage of a general spectral curve. In this case we only need to enter a Spectral Curve Factor. The R/C Building software can use any Spectral Response Curve entered by the user, which is multiplied by a factor defined by the user. If the Spectral Curve represents a real seismic event or a particular earthquake, the factor should be set to 1.0.

Usage: Frequency and Spectral, 2D Advanced

Prepare a 3D model of the building structural system using the R/C Building software program. The model needs to contain all the geometry and all vertical loading.

The model should have a minimum of 4 basic Load Cases, for example:

1. G (dead load)
2. Q (live load)
3. Ex (earthquake force in the X-X direction)
4. Ey (earthquake force in the Y-Y direction)

The model must have a Service Load Combination that will be used to evaluate the mass of the structure and then incorporated in the frequency analysis. For instance, we can define one load combination with 100% Deal Load and 40% Live Load. In this case the load factors are: 1.0G + 0.4Q

On the Model and Solver Settings input panel select:
Load Combination for Floor Mass Centroid: 1.0G + 0.4Q

Select:
Load Case 3: Earthquake in X-X
Load Case 4: Earthquake in Y-Y

Select the number of mode shapes (3 to 6 should be sufficient). Then select the "Advanced 2D" option for evaluating the mode shapes.

Mesh and analyse the model using the Linear Static Analysis option. The stiffness matrix of the structural system is now evaluated, and it is ready to be used in the frequency analysis

Select [Solve > Frequency Analysis]. A text report will open showing the frequency results.

The mode shapes and natural periods shown in the report will now be used for the Spectral Analysis (in the Earthquake Lateral Load input panel).

In the [Input > Lateral Load] window, select basic "Load Case: 3. Ex" and press on the "Earthquake AS 1170.4" button followed by selecting "Dynamic Analysis". Select "X-X direction" and enter all input

parameters and press OK. This will replace the initial set of lateral forces in the table for the X-X direction.
Select "Load Case: 4. Ey" and repeat the procedure outlined in the step above.

The model is now reanalyzed by the 2nd Order Static Analysis option and examine the results can be examined.

The Earthquake load calculator allows usage of a general spectral curve. In this case we only need to enter a Spectral Curve Factor. The R/C Building software can use any Spectral Response Curve entered by the user, which is multiplied by a factor defined by the user. If the Spectral Curve represents a real seismic event or a particular earthquake, the factor should be set to 1.0.

Usage: 3D Frequency and Spectral

Prepare a 3D model of the building structural system using the R/C Building software program. The model needs to contain all the geometry and all vertical loading.

On the Model and Solver input panel choose one Load Combination to be used for Mass.
Usually: 1G+0.4Q or 1G+0.6Q

On the Model and Solver input panel select "Complete 3D" for the Mode Shapes option.

Run the Linear Static or 2nd order Analysis.
Now the stiffness and mass matrices will be saved.

Run Frequency Analysis.
The 3D mode shapes, translation in X-X and Y-Y, global rotation, and the associated natural periods will be evaluated.
Note that M.P.F (Mass Participation Factor) in X-X and Y-Y must be greater than 90%. If M.P.F is less than 90%, then a larger number of mode shapes need to be evaluated.

Frequency analysis results example:

R/C BUILDING, Frequency Analysis 3D
Reading Global Stiffness and Mass Matrices.
Stiffness Matrix Entries: 5518301
Number of Mode Shapes: 9
Period Shift: No
Sturm Sequence Check: Yes
Stiffness and Mass from Linear Static Analysis, 14/05/2008 12:13:35
Mass from Load Combination No: 2
Iterations: 13
Sturm sequence check.
Total number of mode shapes found: 9

	Period, s	Hz	MPFx	MPFy
1	0.9576	1.0	58.3	4.2
2	0.8325	1.2	5.0	61.5
3	0.7661	1.3	3.6	0.9
4	0.3259	3.1	7.4	2.9
5	0.2622	3.8	6.4	10.9
6	0.2142	4.7	5.8	4.9
7	0.1679	6.0	3.4	1.4
8	0.1343	7.4	0.0	0.0
9	0.1341	7.5	0.0	0.0

Total M.P.F. 89.8 86.8

In the example above, the user requested 3 mode shapes to be evaluated. The software will calculate a total of 9 mode shapes, 3 in X-X direction, 3 in Y-Y direction and 3 in rotation. In this example the M.P.F is less than 90%, and generally will need to include larger number of mode shapes in order to get M.P.F greater than 90%.

In this example the largest M.P.F (Mass Participation Factor) in the X-X direction indicates a dominant mode shape in X-X (translation). The dominant mode shape in X-X is Mode No 1, T=0.9576s. The dominant mode shape in Y-Y is Mode No 2, T=0.8325s. The modes with small M.P.F in both directions, such as Mode No 3 are modes in rotation. This will be obvious when the modes are plotted in 3D.

The next step is to run the Spectral Analysis in X-X and select a Basic Load Case where the earthquake action in X-X will be applied. As for the X-X direction, run the Spectral Analysis for the Y-Y direction and select a Basic Load Case where the earthquake action in Y-Y will be applied.
Now the global displacement due to earthquake in X-X and Y-Y are available and stored for the selected Basic Load Cases.

Run Linear Static Analysis or 2nd order.
The integral forces and all other results due to the earthquake actions in X-X and Y-Y will be calculated.

Examine the result as normal.

Note that the Basic Load Cases where the earthquake actions in X-X and Y-Y are applied have to be combined with the other actions with +ve and –ve signs. The results from the Spectral Analysis have no sign and therefore they have to be combined as positive and as negative values with all other actions.

Comparison of the Three Alternative Methods

We have performed a Frequency Analysis of a relatively irregular 3D model shown in the figure above: *Example: 3D Mode Shapes.* The results obtained by all three methods are listed below:

2D Simple, Natural Period
X-X T1: 0.96s T2: 0.43s T3: 0.28 s
Y-Y T1: 0.86s T2: 0.39s T3: 0.25 s

2D Advanced, Natural Period
X-X T1: 0.96s T2: 0.30s T3: 0.17 s
Y-Y T1: 0.86s T2: 0.26s T3: 0.15 s

3D Complete
X-X T1: 0.96s T2: 0.33s T3: 0.17 s
Y-Y T1: 0.83s T2: 0.26s T3: ?

	Period, s	Hz	MPFx	MPFy	
1	0.9576	1.0	58.3	4.2	1. X-X
2	0.8325	1.2	5.0	61.5	1. Y-Y
3	0.7661	1.3	3.6	0.9	Rotation
4	0.3259	3.1	7.4	2.9	2. X-X
5	0.2622	3.8	6.4	10.9	2. Y-Y
6	0.2142	4.7	5.8	4.9	mixed
7	0.1679	6.0	3.4	1.4	3. X-X or mixed
8	0.1343	7.4	0.0	0.0	4. X-X
9	0.1341	7.5	0.0	0.0	5. X-X
Total M.P.F.			89.8	86.8	

We can observe that all three methods provided the same values for the dominant period in X-X direction. While in Y-Y both 2D methods provided larger periods than the 3D approach. This discrepancy is due to the interference of the rotational mode shapes. The dominant natural periods obtained even by the simple methods can be used in the methods where only the dominant mode shapes are considered. The higher periods obtained by the Simple 2D method are not accurate, and they should not be used in the Spectral Analysis. The higher periods obtained by the Advance 2D method are of acceptable accuracy, and they may be used in the Spectral Analysis.
